Rearranging Formulae (Solutions)

Q1, (Jan 2006, Q5)

Make C the subject of the formula $P = \dfrac{C}{C+4}$. [4]

$$P = \frac{C}{C+4} \Rightarrow P(C+4) = C$$
$$\Rightarrow PC + 4P = C$$
$$\Rightarrow PC - C = -4P$$
$$\Rightarrow C(P-1) = -4P$$
$$\Rightarrow \boxed{C = \frac{-4P}{P-1} \text{ or } \frac{4P}{1-P}}$$

Q2, (Jun 2006, Q1)

The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$. Make r the subject of this formula.

[3]

$$V = \frac{1}{3}\pi r^2 h \Rightarrow 3V = \pi r^2 h$$
$$\Rightarrow \frac{3V}{\pi} = r^2 h$$
$$\Rightarrow \frac{3V}{\pi h} = r^2$$
$$\Rightarrow \boxed{r = \pm\sqrt{\frac{3V}{\pi h}}}$$

Q3, (Jan 2007, Q3)

Make a the subject of the equation

$$2a + 5c = af + 7c.$$ [3]

$$2a + 5c = af + 7c \Rightarrow 2a - af = 2c$$
$$\Rightarrow a(2-f) = 2c$$
$$\boxed{\Rightarrow a = \frac{2c}{2-f} \text{ or } \frac{-2c}{f-2}}$$

Q4, (Jun 2007, Q2)

Make t the subject of the formula $s = \frac{1}{2}at^2$. [3]

$$s = \frac{1}{2}at^2 \Rightarrow 2s = at^2 \Rightarrow t^2 = \frac{2s}{a}$$
$$\boxed{\Rightarrow t = \pm\sqrt{\frac{2s}{a}}}$$

Q5, (Jan 2008, Q1)

Make v the subject of the formula $E = \frac{1}{2}mv^2$. [3]

$$E = \frac{1}{2}mv^2 \Rightarrow 2E = mv^2 \Rightarrow \frac{2E}{m} = v^2$$

$$\Rightarrow v = \pm\sqrt{\frac{2E}{m}}$$

Q6, (Jun 2008, Q5)

Make x the subject of the equation $y = \dfrac{x+3}{x-2}$. [4]

$$y = \frac{x+3}{x-2} \Rightarrow y(x-2) = x+3$$

$$\Rightarrow yx - 2y = x + 3$$

$$\Rightarrow yx - x = 3 + 2y$$

$$\Rightarrow x(y-1) = 3 + 2y$$

$$\boxed{\Rightarrow x = \frac{3+2y}{y-1} \text{ or } \frac{-3-2y}{1-y}}$$

Q7, (Jan 2009, Q9)

Rearrange $y + 5 = x(y + 2)$ to make y the subject of the formula. [4]

$$y + 5 = x(y+2) \Rightarrow y + 5 = xy + 2x$$

$$\Rightarrow y - xy = 2x - 5$$

$$\Rightarrow y(1-x) = 2x - 5$$

$$\boxed{\Rightarrow y = \frac{2x-5}{1-x} \text{ or } \frac{5-2x}{x-1}}$$

Q8, (Jun 2009, Q2)

Make a the subject of the formula $s = ut + \frac{1}{2}at^2$. [3]

$$s = ut + \frac{1}{2}at^2 \Rightarrow s - ut = \frac{1}{2}at^2$$

$$\Rightarrow 2(s - ut) = at^2$$

$$\Rightarrow \boxed{a = \frac{2(s-ut)}{t^2}}$$

Q9, (Jan 2010, Q1)

Rearrange the formula $c = \sqrt{\dfrac{a+b}{2}}$ to make a the subject. **[3]**

$$c = \sqrt{\frac{a+b}{2}} \Rightarrow c^2 = \frac{a+b}{2} \Rightarrow 2c^2 = a+b$$

$$\Rightarrow \boxed{a = 2c^2 - b}$$

Q10, (Jun 2010, Q3)

Make y the subject of the formula $a = \dfrac{\sqrt{y}-5}{c}$. **[3]**

$$a = \frac{\sqrt{y}-5}{c} \Rightarrow ac = \sqrt{y} - 5 \Rightarrow ac + 5 = \sqrt{y}$$

$$\Rightarrow \boxed{y = (ac+5)^2}$$

Q11, (Jan 2011, Q5)

The volume V of a cone with base radius r and slant height l is given by the formula

$$V = \tfrac{1}{3}\pi r^2 \sqrt{l^2 - r^2}.$$

Rearrange this formula to make l the subject. **[4]**

$$V = \frac{1}{3}\pi r^2 \sqrt{l^2 - r^2} \Rightarrow 3V = \pi r^2 \sqrt{l^2 - r^2} \Rightarrow \frac{3V}{\pi r^2} = \sqrt{l^2 - r^2}$$

$$\Rightarrow \left(\frac{3V}{\pi r^2}\right)^2 = l^2 - r^2 \Rightarrow l^2 = \left(\frac{3V}{\pi r^2}\right)^2 + r^2$$

$$\Rightarrow \boxed{l = \pm\sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2}}$$

Q12, (Jun 2011, Q8)

Make x the subject of the formula $y = \dfrac{1-2x}{x+3}$. **[4]**

$$y = \frac{1-2x}{x+3} \Rightarrow y(x+3) = 1 - 2x \Rightarrow yx + 3y = 1 - 2x$$

$$\Rightarrow yx + 2x = 1 - 3y \Rightarrow x(y+2) = 1 - 3y$$

$$\Rightarrow \boxed{x = \frac{1-3y}{y+2}}$$

Q13, (Jan 2012, Q6)

Rearrange the following equation to make h the subject.

$$4h + 5 = 9a - ha^2 \qquad [3]$$

$$4h + 5 = 9a - ha^2 \Rightarrow 4h + ha^2 = 9a - 5$$

$$\Rightarrow h(4 + a^2) = 9a - 5$$

$$\Rightarrow \boxed{h = \frac{9a - 5}{4 + a^2}}$$

Q14, (Jun 2012, Q2)

Make b the subject of the following formula.

$$a = \tfrac{2}{3} b^2 c \qquad [3]$$

$$a = \frac{2}{3} b^2 c \Rightarrow 3a = 2b^2c \Rightarrow b^2 = \frac{3a}{2c} \Rightarrow \boxed{b = \pm\sqrt{\frac{3a}{2c}}}$$

Q15, (Jan 2013, Q3)

A circle has diameter d, circumference C, and area A. Starting with the standard formulae for a circle, show that $Cd = kA$, finding the numerical value of k. [3]

$$C = \pi d \Rightarrow Cd = \pi d^2$$

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \Rightarrow 4A = \pi d^2$$

$$\Rightarrow d^2 = \frac{4A}{\pi}$$

$$\therefore \text{Since } Cd = \pi d^2 \Rightarrow Cd = \cancel{\pi}\left(\frac{4A}{\cancel{\pi}}\right)$$

$$\Rightarrow \boxed{Cd = 4A \quad \text{so} \quad k = 4}$$

Q16, (Jan 2013, Q8)

Rearrange the equation $5c + 9t = a(2c + t)$ to make c the subject. [4]

$$5c + 9t = a(2c + t) \Rightarrow 5c + 9t = 2ac + at$$

$$\Rightarrow 5c - 2ac = at - 9t$$

$$\Rightarrow c(5 - 2a) = at - 9t$$

$$\Rightarrow \boxed{c = \frac{at - 9t}{5 - 2a}}$$

Q17, (Jun 2013, Q4)

Rearrange the following formula to make r the subject, where $r > 0$.

$$V = \frac{1}{3}\pi r^2(a+b) \qquad [3]$$

$$V = \frac{1}{3}\pi r^2(a+b) \Rightarrow 3V = \pi r^2(a+b)$$

$$\Rightarrow r^2 = \frac{3V}{\pi(a+b)} \Rightarrow \boxed{r = \pm\sqrt{\frac{3V}{\pi(a+b)}}}$$

Q18, (Jun 2014, Q5)

Make a the subject of $3(a+4) = ac + 5f$. **[4]**

$$3(a+4) = ac + 5f \Rightarrow 3a + 12 = ac + 5f$$

$$\Rightarrow 3a - ac = 5f - 12$$

$$\Rightarrow a(3-c) = 5f - 12$$

$$\Rightarrow \boxed{a = \frac{5f-12}{3-c} \text{ or } \frac{12-5f}{c-3}}$$

Q19, (Jun 2015, Q1)

Make r the subject of the formula $A = \pi r^2(x+y)$, where $r > 0$. **[2]**

$$A = \pi r^2(x+y) \Rightarrow r^2(x+y) = \frac{A}{\pi}$$

$$\Rightarrow r^2 = \frac{A}{\pi(x+y)} \Rightarrow \boxed{r = \sqrt{\frac{A}{\pi(x+y)}}}$$

(Only positive since $r > 0$)

Q20, (Jun 2016, Q4)

You are given that $a = \frac{3c+2a}{2c-5}$. Express a in terms of c. **[4]**

$$a = \frac{3c+2a}{2c-5} \Rightarrow a(2c-5) = 3c + 2a$$

$$\Rightarrow 2ac - 5a = 3c + 2a$$

$$\Rightarrow 2ac - 7a = 3c$$

$$\Rightarrow a(2c-7) = 3c$$

$$\Rightarrow a = \frac{3c}{2c-7}$$

Q21, (Jun 2017, Q6)

Rearrange the formula $r = \sqrt{\frac{V}{a+b}}$ to make b the subject. **[4]**

$$r = \sqrt{\frac{V}{a+b}} \Rightarrow r^2 = \frac{V}{a+b} \Rightarrow r^2(a+b) = V$$

$$\Rightarrow ar^2 + br^2 = V \Rightarrow br^2 = V - ar^2$$

$$\Rightarrow \boxed{b = \frac{V - ar^2}{r^2}}$$

Q22, (Jun 2018, Q4)

For the following equation, express x in terms of y.

$$\frac{x}{3y} = \frac{2x+1}{y+2}$$ **[4]**

$$\frac{x}{3y} = \frac{2x+1}{y+2} \Rightarrow x(y+2) = 3y(2x+1)$$

$$\Rightarrow xy + 2x = 6xy + 3y$$

$$\Rightarrow 2x - 5xy = 3y$$

$$\Rightarrow x(2-5y) = 3y$$

$$\Rightarrow \boxed{x = \frac{3y}{2-5y}}$$